Does Special Relativity Have Limits of Applicability in the Domain of Very Large Energies?

L.Ya.Kobelev
Department of Physics, Urals State University
Av. Lenina, 51, Ekaterinburg 620083, Russia
E-mail: leonid.kobelev@usu.ru

We have shown in the paper that for time with fractional dimensions (multifractal time theory) there are small domain of velocities v near $v = c$ where SR must be replaced by fractal theory of almost inertial system that doe's not contains an infinity and permits moving with arbitrary velocities.

01.30.Tt, 05.45, 64.60.A; 00.89.98.02.90.+p.

I. INTRODUCTION

The special relativity theory (SR) is one of the physics theories that are compose the base of the modern physics, it has well experimental foundation in the large area of the reached velocities and energies, is the working theory of a modern physics and widely use in a science and technique. Nevertheless, as any physical theory created by men, SR has a boundaries of applicability (the inertial systems of reference). As far as we know a problem of energy boundaries SR for any moving body was not analyzed in detail. At the same time the tends of energy of a moving body when it's velocity reaches the value of speed of light to infinity calls doubts in applicability SR in this area of energies, as the occurrence of infinity in the physical theories always testifies about their interior deficiencies. The purpose of the paper is presenting the theory of almost inertial systems in the space with multifractal time (I believe that time and space of our world may have multifractal characteristics) in which the motions of bodies with arbitrary velocities are possible and there are no energy infinity. At the same time if the energies reached by a body, as a result of it's motion, are smaller than $E_0 10^3 sec^{1/2}\ t^{-1/2}$ (t-time of acceleration of a particle up to such energies, E_0- a rest energy) all results of the theory coincide with results of SR and thus do not contradict experiment. The differences between our theory and SR appear only if energy of moving body exceeds the energy $E_0 10^3 sec^{1/2} t^{-1/2}$. The theory is based on using, but as a good approach, of a principle of the constancy speed of light, an invariance of modified Galilean and Lorentz transformation laws. This theory is not a generalization of SR, because any SR generalizations in the domain of validity SR (inertial systems) are absurd. This theory describes relative movements only in the almost inertial systems, and thus does not contradict SR, thou coincide with it for case of inertial systems. For constructing such theory it is necessary to refuse from the rigorous realization of the SR postulates: a homogeneous of space and time, the constancy of speed of light, the Galilean relativity principle. In an inhomogeneous space and time, if the inhomogeneous are small, the motion of bodies will be almost inertial, and velocity of light is almost stationary value. This paper contains the example of the theory based on the time and the space with fractional dimensions (FD) [1] . This theory use for description of time and space characteristics the ideas of fractal geometry. The values FD are a little bit distinguished from the integer dimensions (the theory of multifractal time and space is given in [1]. In fractal theory the motions of particles with arbitrary velocities are permissible, the speed of light is almost independent from the velocity of lights sources. For example, on the surface of Earth the differences of value of speed of light under change moving direction v by $-v$ consist $\sim 2v/c 10^{-6} t$. The theory almost coincides with SR, for velocities which are lesser than speed of light but does not includes singularities at $v = c$.

II. MULTIFRACTAL TIME

Following [1], we will consider both time and space as the initial real material fields existing in the world and generating all other physical fields. Assume that every of them consists of a continuous, but not differentiable bounded set of small elements (elementary intervals, further treated as "points"). Consider the set of small time elements S_t. Let time be defined on multifractal subsets of such elements, defined on certain measure carrier $\mathcal{R}^\backslash$. Each element of these subsets (or "points") is characterized by the fractional (fractal) dimension (FD) $d_t(\mathbf{r}(t), t)$ and for different elements FD are different. In this case the classical mathematical calculus or fractional (say, Riemann - Liouville) calculus [2] can not be applied to describe a small changes of a continuous function of physical values $f(t)$, defined on time subsets S_t, because the fractional exponent depends on the coordinates and time. Therefore, we have to introduce integral functionals (both left-sided and right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets (see [1]). Actually, these functionals are simple

and natural generalization of the Riemann-Liouville fractional derivatives and integrals:

$$D^d_{+,t}f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \quad (1)$$

$$D^d_{-,t}f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t'-t)^{d(t')-n+1}} \quad (2)$$

where $\Gamma(x)$ is Euler's gamma function, and a and b are some constants from $[0, \infty)$. In these definitions, as usually, $n = \{d\} + 1$, where $\{d\}$ is the integer part of d if $d \geq 0$ (i.e. $n - 1 \leq d < n$) and $n = 0$ for $d < 0$. If $d = const$, the generalized fractional derivatives (GFD) (1)-(2) coincide with the Riemann - Liouville fractional derivatives ($d \geq 0$) or fractional integrals ($d < 0$). When $d = n+\varepsilon(t)$, $\varepsilon(t) \to 0$, GFD can be represented by means of integer derivatives and integrals. For $n = 1$, that is, $d = 1 + \varepsilon$, $|\varepsilon| << 1$ it is possible to obtain:

$$D^{1+\varepsilon}_{+,t}f(t) \approx \frac{\partial}{\partial t}f(t) + a\frac{\partial}{\partial t}\left[\varepsilon(r(t), t)f(t)\right] \quad (3)$$

where a is constant and defined by the choice of the rules of regularization of integrals (1)-(2) (for more detailed see [1]). The selection of the rule of regularization that gives a real additives for usual derivative in (3) yield $a = 0.5$ for $d < 1$ and $a = 1.077$ for $d > 1$ [1]. The functions under integral sign in (1)-(2) we consider as the generalized functions defined on the set of the finite functions [3]. The notions of GFD, similar to (1)-(2), can also be defined and for the space variables $\mathbf{r}$.

The definitions of GFD (2)-(2) are formal until the connections between fractal dimensions of time $d_t(\mathbf{r}(t), t)$ and certain characteristics of physical fields (say, potentials $\Phi_i(\mathbf{r}(t), t)$, $i = 1, 2, ..$) or densities of Lagrangians L_i) are determined. Following [1], we define this connection by the relation

$$d_t(\mathbf{r}(t), t) = 1 + \sum_i \beta_i L_i(\Phi_i(\mathbf{r}(t), t)) \quad (4)$$

where L_i are densities of energy of physical fields, β_i are dimensional constants with physical dimension of $[L_i]^{-1}$ (it is worth to choose β'_i in the form $\beta'_i = a^{-1}\beta_i$ for the sake of independence from regularization constant). The definition of time as the system of subsets and definition the FD d (see 4) connects the value of fractional (fractal) dimension $d_t(r(t), t)$ with each time instant t. The latter depends both on time t and coordinates $\mathbf{r}$. If $d_t = 1$ (an absence of physical fields) the set of time has topological dimension equal to unity. The multifractal model of time allows, as will be shown below, to consider the divergence of energy of masses moving with speed of light in the SR theory as the result of the requirement of rigorous validity of the laws pointed out in the beginning of this paper in the presence of physical fields (in our theory there are approximate fulfillment of these laws).

III. THE PRINCIPLE OF THE SPEED OF LIGHT INVARIANCE

Because of the non-uniformity of time in our multifractal model, the speed of light, just as in the general relativity theory, depends on potentials of physical fields that define the fractal dimensions of time $d_t(\mathbf{r}(t), t)$ (see (4)). If fractal dimension $d_t(\mathbf{r}(t), t)$ is close enough to unity ($d_t(r(t), t) = 1 + \varepsilon$, $|\varepsilon| << 1$), the difference of the speed of light in moving (with velocity v along the x axis) and fixed frame of reference will be small. In the systems that move with respect to each other with almost constant velocity (stationary velocities do not exist in the mathematical theory based on the definitions of GFD (1) - (2)) the speed of light can not be taken as a fundamental constant. In the multifractal time theory the principle of the speed of light invariance can be considered only as approximate. But if ε is small, it allows to consider a nonlinear coordinates transformations from the fixed frame to the moving frame (replacing the Galilean transformations in non-uniform time and space), as close to linear (weakly nonlinear) transformations and, thus, makes it possible to preserve the conservation laws, and all the invariant's of the Minkowski space, as the approximate laws. Then the way of reasoning and argumentation accepted in SR theory (see for example, ([4])) can also remains valid. Designating the coordinates in the moving and fixed frames of reference through x' and x, accordingly, we write down

$$\begin{aligned} x' &= \alpha(t, x)[x - v(x, t)t(x(t), t] \\ x &= \alpha'(t, x)[x' + v'(x'(t'), t'), t'(x'(t'), t') \end{aligned} \quad (5)$$

In (5) $\alpha \neq \alpha'$ and the velocities v' and v (as well as t and t') are not equal (it follows from the inhomogeneous of multifractal time). Place clocks in origins of both the frame of references and let the light signal be emitted in the moment, when the origins of the fixed and moving frames coincide in space and time at the instant $t^1 = t = 0$ and in points $x' = x = 0$. The propagation of light in moving and fixed frames of reference is then determined by equations

$$x' = c't' \;\; x = ct \quad (6)$$

These equations characterize the propagation of light in both of the frames of reference at every moment. Due to the time inhomogeneous $c' \neq c$, but since $|\varepsilon << 1|$ the difference between velocities of light in the two frames of reference will be small. For this case we can neglect by the differences between α' and α and, for different frames of reference write the expressions for velocities of light (using (3) to define velocity (denote $f(t) = x$, $dx/dt = c_0$)). Thus we obtain

$$c = D^{1+\varepsilon}_{+,t}x = c_0(1 - \varepsilon) - \frac{d\varepsilon}{dt}x \quad (7)$$

$$c' = D^{1+\varepsilon'}_{+,t}x' = c_0(1 - \varepsilon) - \frac{d\varepsilon}{dt}x' \quad (8)$$

$$c_1 = c_0(1-\varepsilon) - \frac{d\varepsilon}{dt}x' \tag{9}$$

$$c_1' = c_0(1-\varepsilon) - \frac{d\varepsilon}{dt}x \tag{10}$$

The equalities (9) and (10) appear in our model of multifractal time as the result of the statement, that in this model all the frames of reference are absolute frames of reference (because of the real nature of the time field) and the speed of light depends on the state of frames: if the frame of reference is a moving or a fixed one, if the object under consideration in this frame moves or not. This dependence disappears only when $\varepsilon = 0$. Before substitution the relations (5) in the equalities (7) - (10) (with $\alpha' \approx \alpha$) it is necessary to find out how $d\varepsilon/dt$ depends on α. Using for this purpose equation (4) we obtain:

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon}{d\mathbf{r}}\mathbf{v} \approx -\sum_i \beta_i(\mathbf{F}_i\mathbf{v} + \frac{\partial L_i}{\partial t}) \tag{11}$$

where $\mathbf{F}_i = \frac{dL_i}{d\mathbf{r}}$. As the forces for moving frames of reference are proportional to α we get (for the case when there is no explicit dependence of L_i on time)

$$\frac{d\varepsilon}{dt} \approx -\sum_i \beta_i \mathbf{F}_{0i}\mathbf{v}\alpha \tag{12}$$

where F_{0i} are the corresponding forces at zero velocity. Multiplying (7) - (10) on the corresponding times t, t', t_1, t_1' yields the following expressions

$$c't' = c_0 t\left[1 + \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1-\frac{v}{c})\right] \tag{13}$$

$$ct = c_0 t'\left[1 + \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1-\frac{v}{c})\right] \tag{14}$$

$$c_1't_1' = c_0 t_1\left[1 - \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1-\frac{v}{c})\right] \tag{15}$$

$$c_1 t_1 = c_0 t_1'\left[1 - \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1-\frac{v}{c})\right] \tag{16}$$

Since in our model the motion and frames of reference are absolute, the times t_1 and t_1' correspond to the cases, when the moving and fixed frames of reference exchange their roles - the moving one becomes fixed and vice versa. These times coincide only when $\varepsilon = 0$. The times in square brackets, as well as the velocities, are taken to equal, because the terms containing them are small as compared to unity. The principle of invariance of the velocity of light for transition between the moving and fixed frames of reference in multifractal time model is approximate (though quite natural, because the frames of reference are absolute frames of reference). Taking into account (5), the relations (13) - (16) take the forms

$$c't' = c\alpha t(1-\frac{v}{c}), \quad c_1't_1' = c\alpha t_1(1-\frac{v}{c}) \tag{17}$$

$$ct = c\alpha t'(1+\frac{v}{c}), \quad c_1 t_1 = c\alpha t'(1+\frac{v}{c}) \tag{18}$$

Once again we note, that the four equations for $c_1't_1'$ and c_1t_1, instead of the two equations in special relativity, appear as the consequences of the absolute character of the motion and frames of reference in the model of multifractal time. In the right-hand side of (17) - (18) the dependence of velocity of light on fractal dimensions of time is not taken into account (just as in the equations (13) - (16)). Actually, this dependence leads to pretty unwieldy expressions. But if we retain only the terms that depend on $\beta = \sqrt{|1 - v^2/c^2|}$ or a_0 and neglect non-essential terms containing the products $\beta\alpha_0$, utilizing (13) - (16) after the multiplication of the four equalities (17) - (18), we receive the following equation for α (it satisfies to all four equations):

$$4a_0^4\beta^4\alpha^8 - 4a_0^2\alpha^4 + 1 = \beta^4\alpha^4 + 4a_0^4\beta^4\alpha^8 \tag{19}$$

where

$$\beta = \sqrt{\left|1 - \frac{v^2}{c^2}\right|} \tag{20}$$

$$a_0 = \sum_i \beta_i F_{0i}\frac{v}{c}ct \tag{21}$$

From (19) follows

$$\alpha_1 \equiv \beta_*^{-1} = \frac{1}{\sqrt[4]{\beta^4 + 4a_0^2}} \tag{22}$$

The solutions $\alpha_{2,3,4}$ are given by $\alpha_2 = -\alpha_1$, $\alpha_{3,4} = \pm i\alpha$. Applicability of above obtained results is restricted by requirement $|\varepsilon| \ll 1$

IV. LORENTZ TRANSFORMATIONS AND TRANSFORMATIONS OF LENGTH AND TIME IN MULTIFRACTAL TIME MODEL

The Lorentz transformations, as well as transformations of coordinate frames of reference, in the multifractal model of time are nonlinear due to the dependence of the fractional dimensions of time $d_t(\mathbf{r}, t)$ on coordinates and time. Since the nonlinear corrections to Lorentz transformation rules are very small for $\varepsilon \ll 1$, we shall take into account only the corrections that eliminate the singularity at the velocity $v = c$. It yields in the replacement of the factor β^{-1} in Lorentz transformations by the modified factor $\alpha = 1/\beta^*$ given by (22). The Lorentz

transformation rules (for the motion along the x axis) take the form

$$x' = \frac{1}{\beta^*}(x - vt),\ \ t'\frac{1}{\beta^*}(t - x\frac{v}{c^2}) \tag{23}$$

In the equations (22) and (23) the velocities v and c weakly depend on x and t and their contribution to the singular terms are small. Hence, we can neglect by this dependence. The transformations from fixed system to moving system are almost orthogonal (for $\varepsilon \ll 1$), and the squares of almost four-dimensional the energy-momentum vectors of Minkowski space vary under the coordinates transformations very slightly (i.e. they are almost invariant). Then it is possible to neglect the correction terms of order about $O(\varepsilon, \dot{\varepsilon})$, which, for not equal to infinity variables, are very small too. From (22) - (23) the possibility of arbitrary velocity motion of bodies with nonzero rest mass follows. With the corrections of to order $O(\varepsilon, \dot{\varepsilon})$ in nonsingular terms being neglected, the momentum and energy of a body with a nonzero rest mass in the frame of reference moving along the x axis ($E_0 = m_0c^2$) equal to

$$p = \frac{1}{\beta^*}m_0v = \frac{m_0v}{\sqrt[4]{\beta^4 + 4a_0^2}},\ \ E = E_0\sqrt{\frac{v^2c^{-2}}{\sqrt{\beta^4 + 4a_0^2}} + 1} \tag{24}$$

The energy of such a body reaches its maximal value at $v = c$ and is equal then $E_{v=c} \approx E_0/\sqrt{2\alpha_0}$. When $v \to \infty$ the energy is finite an tends to $E_0\sqrt{2}$. For $v \leq c$ the total energy of a body is represented by the expression

$$E \cong \frac{E_0}{\sqrt[4]{\beta^4 + 4a_0^2}} = mc^2,\ \ m = \frac{m_0}{\beta^*} \tag{25}$$

For $v \geq c$, total energy, defined by (24), is given by

$$m = \beta^{*-1}m_0\sqrt{1 + \beta^{*2} + \sqrt{\beta^{*4} - 4a_0^2}} \tag{26}$$

If we take into account only the gravitational field of Earth (here, as in ([5]),the gravitational field is a real field) and neglect by the influences of all the other fields), the parameter $a_0(t)$ can be estimated as $a_0 = r_0R^{-3}x_Ect$, where r_0 is the gravitational radius of Earth, r is the distance from the Earth's surface to its center ($\varepsilon = 0.5\beta_g\Phi_g$, $\beta_g = 2c^{-2}$, $x_E \sim r_0$, $v = c$). For energy maximum we get $E_{max} \sim E_0 \cdot 10^3t^{-0.5}sec^{-0.5}$. If we take into account only the constant electric field with electric strange E then parameter a_0 has the value:$a_0 = \frac{eE}{Mc^2}ct$ where Mc^2 is the rest energy of charges originated the strange E and t is the time of acceleration of particle. Contraction of lengths and the retardation of time in moving frames of reference in the model of multifractal time are also have several peculiarities. Let l and t be the length and time interval in a fixed frame of reference. In a moving frame

$$l' = \beta^*l,\ \ t' = \beta^*t \tag{27}$$

Thus, there exist the maximal contraction of length when the body's velocity equals the speed of light, but length is not equal to zero. With the further increasing of velocity (if it is possible to fulfill some requirements for a motion in this region with constant velocity without radiating), the length of a body begins to grow and at infinitely large velocity is also infinite. The retardation of time, from the point of view of the observer in the fixed system (maximal retardation equals to $t' = t\sqrt{2a_0}$) is replaced, with the further increase of velocity over the speed of light, by acceleration of a flow of time ($t \to 0$ when $v \to \infty$).

The rule for velocities transformation keeps its form, but β is replaced by β^*

$$u_x = \frac{u_x' + v}{1 + \frac{u_x'v}{c^2}},\ u_y = \frac{u_y'\beta^*}{1 + \frac{u_y'v}{c^2}},\ u_z = \frac{u_z'\beta^*}{1 + \frac{u_z'v}{c^2}} \tag{28}$$

Since there is no law that prohibits velocities greater than that of light, the velocities in (28) can also exceed the speed of light. The electrodynamics of moving media in the model of multifractal time can be obtained, in most cases, by the substitution $\beta \to \beta^*$.

V. NEWTON EQUATION FOR RELATIVISTIC PARTICLE

The Newton relativistic equation for particle with velocities $v \leq c$ has form

$$\frac{\partial}{\partial t}(m\mathbf{v}) = \frac{\partial}{\partial t}\left(\frac{m_0\mathbf{v}}{\sqrt[4]{(1 - \frac{v_2}{c_2})^2 + 4a_0^2}}\right) = e\mathbf{E} \tag{29}$$

For $v \geq c$ the mass m in equation (29) is determined by (26). If $\mathbf{E} = const.$ (29) gives for $v = c$ possibility to find (neglecting by radiation of charge) minimum for the time t_0 that is necessary for receiving by particles the velocity equal c

$$\frac{m_0c}{\sqrt{\frac{2eEct_0}{Mc^2}}} = eEt_0 \tag{30}$$

or

$$t_0 = \sqrt[3]{\frac{Mc^2}{2E_0}}\ \ \frac{E_0}{eEc} \tag{31}$$

The t_0 defined by (31) gives only an order of the value t that is necessary for receiving by particle velocity $v = c$. The maximum energy at $v = c$ may be written now as (we introduce the value $\alpha < 1$ for describing the radiation energy losses)

$$E_{max} = E_0\sqrt[3]{\frac{Mc^2}{2E_0\alpha}} \tag{32}$$

The values of α and electric strange E are determined by construction of accelerators and conditions of their work regimes.

VI. CONCLUSIONS

The theory of relative motions in almost inertial systems based on the multifractal time theory [1] and constructed in this paper gives in the new describing for characteristics (energy, momentum, mass and so on) of moving bodies. The main results are:a) the possibility of moving with arbitrary velocities without appearance of infinitum energy and imaginary mass; b) existence of maximum energy if $v = c$; c) possibility of experimental verification the main results of the theory. This theory describes open systems (theory of open systems see in [6]). The theory coincides with SR after transition to inertial systems (if neglect by the fractional dimensions of time) or almost coincides (the differences are non-essential) for velocities $v < c$. The movement of bodies with velocities that exceed the speed of light is accompanied by a series of physical effect's which can be found experimentally (these effects was considered in the separate papers ([1]) in more details). It is necessary for verification of the theory to receive the particle's with energy $E \sim \frac{E_0}{\sqrt{\frac{2eE}{Mc^2}ct_0}}$ where t_0 defined by (31). If take into account the radiation losses of energy, it will be enough to receive at the intervals of time acceleration of the particle about one second the energies of order $E \sim E_0 10^3$.

---

[1] Kobelev L.Ya. *Fractal Theory Time and Space*, Ekaterinburg: Konross, 1999, 136p.(in Russian);*Fractal Theory Time and Space* /Kobelev L.Ya.// Ural State Univ., Ekaterinburg, 1998.-158p.-Bibliogr.51Nam.-Rus.-Dep.v VINITY 22.01.99,189-B99 (in Russian.); Multyfractality of Time and Special Theory of Relativity / Kobelev L.Ya.// Ural State Univ., Ekaterinburg, 1999.-21p.-Bibliogr.14Ref.-Rus.-Dep.v VINITY 19.08.99, 2677-B99.01.99,(in Rus.); Kobelev L.Ya./ Dep. v VINITI. Ekaterinburg. 20.10.99.3128-B99; Kobelev L.Ya., What Dimensions Do the Time and Space Have: Integer or Fractional? arXiv:physics/0001035; Kobelev L.Ya., Can a Particle's Velocity Exceeds the Speed of Light in the Empty Space? arXiv:gr-qc/0001042; Kobelev L.Ya.,Physical Consequences of Moving Faster than Light in Empty Space, arXiv:gr-qc /0001043 ;Kobelev L.Ya., Multifractality of Time and Space, Covariant Derivatives and Gauge Invariance,arXiv:hep-th/ 0002005; Kobelev L.Ya.,Generalized Riemann -Liouville Fractional Derivatives for Multifractal Sets, arXiv:math. CA/0002008,; Kobelev L.Ya., The Multifractal Time and Irreversibility in Dynamic Systems, arXiv:physics/0002002; Kobelev L.Ya., Is it Possible to Transfer an Information with the Velocities Exceeding Speed of Light in Empty Space?,arXiv: physics/ 0002003; Kobelev L.Ya., Maxwell Equation, Shroedinger Equation, Dirac Equation, Einstein Equation Defined on the Multi fractal Sets of the Time and the Space, arXiv:gr-qc/0002003

[2] S.G.Samko , A.A.Kilbas , O.I.Marichev, *Fractional integrals and derivatives - theory and applications* (Gordon and Breach, New York, 1993)

[3] I.M.Gelfand, G.E.Shilov, *Generalized functions* (Academic Press, New York, 1964)

[4] Matveev A.N.*Mechanics and theory of relativity* (Higher School, Moscow, 1986)(in Russian)

[5] Logunov A.A.,Mestvirichvili M.A.,Theoretical and Mathematical Physics,1997, v.110, p.1-20 (in Russian)

[6] Klimontovich Yu.L.*Statistical theory of open systems* (Kluwer, Dordrecht, 1995);Klimontovich Yu.L.,*Statistical theory of open systems 2*,(Moscow: Yanus,1999, 439p)(in Russian)